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PROBLEMS IN OPTIMAL FILTERING AND STOCHASTIC CONTROL.(U)
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PROBLEMS IN OPTIMAL FILTERING AND STOCHASTIC CONTROL,

(15) AFOSR-77-3286

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OUTLINE OF REPORT

I. APPROXIMATION TECHNIQUES IN STOCHASTIC CONTROL PROBLEMS

In this research we continue our investigations of approximation techniques for a wide class of discrete and continuous time stochastic control problems. Emphasis is placed on the development and theoretical justification of techniques which yield computationally tractable algorithms that answer the following:

(1) approximations to the optimal cost and the cost of using a particular control.

(2) approximations to the optimal control.

(3) evaluation of the relative performance of two controls.

(4) estimates for the deterioration in system performance due to the failure to observe system components.

Small Noise Problems in Continuous Time

In many control problems the noise entering the system is of low intensity reflecting its role as a nuisance parameter. Recognizing the extreme complexity of solving stochastic control problems, one approach has been to solve "approximately" the stochastic control problem in terms of quantities computable from the optimal solution to the corresponding deterministic control problem which results when the noise is absent.

The general procedure in this approximation technique is to first establish expansions of the optimal cost and control in powers of the noise coefficient. The expansions then suggest appropriate forms for nearly optimal controls and numerical techniques for determining them

The theoretical portion of this research was first treated successfully by Fleming [4]. The numerical algorithms suggested by the theorems were developed and employed on some two dimensional examples by this author in Holland [9].

A partial solution to the open loop control problem was developed by this author in Holland [9], [12]. In those papers we were able to theoretically establish an expansion of the optimal cost in powers of the noise coefficient. However, we were able only to derive and theoretically justify an approximation technique for the optimal control in the restricted case in which each open loop control generates a nondegenerate Gaussian process.

Motivated by an attempt to remove the restriction in these previous results, we recently considered in [17] more general open loop control problems in which each open loop control does not necessarily generate a nondegenerate Gaussian process. An approximation technique was developed that has the advantage that one finds approximately optimal controls simultaneously for all sufficiently small noise coefficients. This scheme produces a control which performs better than the use of the optimal deterministic control problem in the stochastic control problem. Moreover as we show in [17], this scheme is superior to and does not agree with the standard secondary accessory control problem.

In this research we intend to continue our investigations into the numerical methods suggested by the above schemes. The numerical method requires the calculation of a generalized linear regulator problem which can be solved numerically. Attempts will be made at order reduction of the associated problem.

In addition, we intend to look at extensions of the above techniques to sampled-data problems.

Finally, we are interested in a class of completely observable stationary control problems. Expansions in powers of the noise coefficient for using a fixed control have been developed by this author in [11], [14].

Qualitative dependence of stochastic control problems on noise

An area of interest has been the study of the qualitative behavior of the optimal control and cost of stochastic control problems. One is not sure the manner in which the introduction of stochastic effects affects the behavior of the optimal stochastic control and cost. We have done some numerical work in [9] for the completely observable problem on the perturbation of switching surfaces but generalizations are unable to be made.

We recently discovered that a certain class of stationary control problems have the alternative interpretation of finding the principal eigenvalue of second order elliptic, not necessarily self-adjoint, partial differential equations. From the knowledge of the principal eigenvalue and eigenfunction one can then deduce the behavior of the optimal stochastic control and cost. This then gives a class of stochastic control problems for which one has an alternative way of studying the qualitative effects of noise. We intend to continue this investigation further.

Fleming [6] (and with Tsai [3]) have been using stochastic control methods which are similar to ours to answer interesting

probabilistic questions on the physically significant exit problem. This gives a second class of problems to study noise effects.

Lastly, Benes [1] has been examining classes of problems for which one can compute explicitly the optimal stochastic control.

II. PROBLEMS IN OPTIMAL NONLINEAR FILTERING AND CONTROL

Stationary discrete time control problems

Discrete time stochastic control problems have been thoroughly considered. However, most prior work considers the case in which the controller remembers all previously obtained information. This has been called the classical information pattern by Witsenhausen [24]. We intend to investigate the stationary control case where the controller has only partial observations of the system state and no memory. This emphasis is motivated by systems in which it is too difficult or expensive to observe all system components, and systems in which it is difficult to implement controls using past information.

Under certain reasonable assumptions we have succeeded in reducing the optimization problem to a problem in nonlinear programming [13]. We have also constructed examples to show that one can do better by using randomized controls than by only using nonrandomized controls of the current observed data. In problems of this type with complete observations it is known that the optimization problem can be treated as a problem in linear programming [22] and that the controller cannot do better by considering randomized controls [2].

Continuous time filtering problems

In this part of the research we seek a computationally convenient technique for solving the filtering and prediction problems for a class of nonlinear stochastic differential equations subject to partial observations at discrete time points. The

applications of the technique described below to trajectory estimation are apparent.

Let there be given q functions $f_1, f_2, \ldots f_q$ which are known to us. Suppose for some $i \in \{1,2,\ldots q\}$, unknown to us, the state of the process is evolving according to the vector stochastic differential equations

$$dx = f_i(t,x)dt + g(t,x)dw, \quad x(o) = x.$$

At the discrete times $t_1, \ldots t_p$, which are known in advance, an intermediate observer (usually a machine) receives noise corrupted observations of the stochastic process $x(t)$. These observations $y(t)$ satisfy the stochastic differential equations

$$dy = H(t,x,y)dt + \tilde{g}(t,x,y)dw, \quad y(o) = 0.$$

Suppose that $y(t) \in R^m$ for some m. Assume there exists known pairwise disjoint sets B_i, $i = 1,\ldots,m$, $\cup B_i = R^m$. Then at each time t_j, $j = 1,2,\ldots p$, we receive from the intermediate observer only the information as to which of the events $y(t_j) \in B_i$ has occurred.

Our first problem is to determine for each function f_i, the probability that the function f_i is being used given available information.

Our second prediction problem is the following: Given our information at times $t = t_1, t_2, \ldots, t_p$, and the function f_i, determine the best prediction in mean square of some function $h(x(T))$ of the process, $T > t_p$.

Let us discuss problem 2 which we have recently solved theoretically. For each possible information set, we must solve a coupled set of second order partial differential equations. Since the boundary conditions are of Cauchy type, they can be solved numerically. There are $m \times p$ information sets, each requiring the solving of the coupled set of partial differential equations. Although these problems appear complex, they have the important practical advantage that they can be completely precomputed and do not need to be solved in real-time.

In this research we intend to consider the problems discussed above looking at both the theoretical solution and effective computational methods that can be developed from the theoretical solutions.

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III. SUMMARY OF RESEARCH ACCOMPLISHMENTS

This section summarizes progress made under AFOSR Grant 77-3286 Problems in Optimal Filtering and Stochastic Control. The research effort was concentrated in the following areas:

(i) Approximation Procedures for Open Loop Stochastic Control Problems.

(ii) Stationary Stochastic Control and Eigenvalue Problems.

(iii) Long-Term Effects of Noise on Dynamical Systems.

(iv) Asymptotic properties of Nonlinear-Diffusion Equations.

The first three areas are relevant to the control of stochastic systems. The fourth area deals with stability properties and the long term behavior of systems described by nonlinear partial differential equations. No progress was made in the area of nonlinear filtering, although it should be remarked that a transformation in area (ii) above was developed and utilized independently by W. Fleming, also under AFOSR support, to study the unnormalized Zakai equation arising in nonlinear filtering.

Progress in each of the above areas is described in the following pages.

Approximation Procedures for Open Loop Stochastic Control Problems

An Approximation technique for small noise open loop control problems, Optimal Control Applications and Methods 2(1981) 89-94.

Nonlinear perturbations of the open loop stochastic linear regulator problem, working paper.

A major part of our research effort has concentrated on the development of approximation techniques for open loop stochastic control problems. The basic theory for the optimal control of Markov diffusion processes has been the subject of extensive research over the past decade (see Fleming-Rishel (2)). There remains the problem of developing numerical procedures for computing the optimal control, or if that is not possible, approximately optimal controls. If an approximately optimal control is employed, one wants to know the deterioration in performance obtained by using this approximately optimal control.

While most deterministic control problems can be solved numerically, the general problem of computing the optimal stochastic control is impossible with present-day computers. The major exception is the stochastic linear regulator problem both in the closed loop (completely observable) and the open loop cases.

One is thus forced to develop procedures for computing approximately optimal controls for most stochastic control problems. The basic idea has been to solve stochastic control problems which are in some

sense a small perturbation of a control problem for which the optimal control can be computed explicitly. As was mentioned earlier, two such classes of problems exist:

a. deterministic control problems.

b. stochastic linear regulator problems.

Both of these classes of problems have been exploited for the completely observable case. First, Fleming (1) developed the mathematical theory for solving approximately stochastic control problems in case the intensity of the noise entering the system is small. He suggested a numerical method for computing the optimal control in terms of quantities computable with knowledge of the optimal solution of the corresponding deterministic control problem which is obtained when the small noise term is absent. Holland developed the numerical procedure in (3). This numerical procedure has been adapted to a model of a problem of suboptimal control of statellite momenta in a noisy environment (see (4)).

Second, Tsai (5) developed the theory and approximation procedure for the approximate solution of stochastic control problems which arise by perturbing the stochastic linear regulator problem through an additive term with a small parameter b in the drift coefficient of the unperturbed dynamical system. His results show that an approximate solution can be computed in powers b, b^2 b^3, ... of the small parameter b. The coefficients of b, b^2, b^3, ... are computable from a knowledge of the computable solution to the stochastic linear regulator problem. Moreover, the results there give an estimate of the extra cost obtained by using the approximate contol.

In this research we have attempted to duplicate these previous techniques for the stochastic open loop control problem. Open loop controls are easier to implement than closed loop controls since they do not involve real time processing of data. However, this ease of implementation appears to be partially mitigated by the extra complexity in computing approximate controls for case b. above.

First, we treated the open loop problems in case the noise entering the system is small. This problem is the analogue of the completely observable problem treated by Fleming (1). Although there is some analogy with the results of the completely observable case, the methods required in the open loop case are quite different.

Our prior theoretical work in this area had suggested that the optimal control should be expandable in powers of the noise intensity. An approximation technique was thus sought that would calculate "best" controls of a certain form. If an expansion of the optimal control were valid, the approximate control would match the expansion. We were successful with the development of an approximation scheme for these problems. The scheme requires the solution of a generalized linear regulator problem (of determistic type) which is solved easily numerically. The numerical method is given and an example illustrating the efficiency of the method is also presented in the first paper listed in the work for this section.

Recently, we have attempted to develop an approximation technique for the approximate solution for the problem of nonlinear perturbations of

open loop stochastic linear regulator problem (case b above). The problem is as follows:

For each open loop control $u(t)$, the state $x(t)$ evolves according to the Ito stochastic differential equation

$$dx= Ax+Bu+bg(x)\ dt + m(t)dw(t),\ x(0)=x_0$$

where A, B can depend upon t, and b is a small parameter reflecting the intensity of the noise. The cost of using the control $u(t)$ is given by

$$E \int_0^T x(t)M(t)x(t)+ u(t)N(t)u(t)\ dt.$$

The problem is to choose the open loop control u which minimizes the cost.

This problem is the direct analogue of the completely observable problem treated by Tsai (5). His methods do not apply, however, to the open loop case. It was shown for the completely observable case that the optimal feedback control Y^b in the b problem could be expanded as

$$Y^b = Y^0+ bZ_1+b^2Z_2+\ldots+b^kZ_k +o(b^k)$$

for any positive integer k where the coefficients $Z_i(t,x)$ could be computed with a knowledge of the optimal solution of the linear regulator problem. Moreover, his results give an estimate of the performance obtained by using as an approximate control

$$Y^0+bZ_1+\ldots+b^kZ_k$$

for any positive integer k.

We were interested in a duplication of the analogous results for the open loop control problem. Thus, if u^b is the optimal open loop control, one would want to establish the above expansion with Y^b, Y^0 replaced by u^b, u^0 and the functions Z_i replaced by appropriate functions. We have been unable to develop such an approximation scheme.

However, we have shown the following. Suppose we use any control of the form u^0+bV for some function V. Then the cost of using that control can be expanded as

$$C + b\, E \int_0^T S_x(t,\, x^0(t))g(x^0(t))\, dt \quad , C = S(0,X_0)$$

where S(s,x) is the cost obtained in the open loop linear regulator control problem (b=0) starting at (s,x) and using the optimal open loop control for the linear regulator problem corresponding to initial data $(0,x_0)$. The process $x^0(t)$ represents the solution to the above Ito stochastic differential equation with b=0 and use of the optimal linear regulator control as the control function in the system dynamics.

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Stationary Stochastic Control and Eigenvalue Problems:

The principal eigenvalue for linear second order elliptic equations with natural boundary conditions in Stochastic Analysis, 139-152. A. Friedman (Editor), Academic Press (1978).

A minimum principle for the smallest eigenvalue for second order linear elliptic equations with natural boundary conditions, Comm. Pure Appl. Math. 31(1978), 509-519.

We have been able to derive a new characterization of the principal eigenvalue for second order linear elliptic partial differential equations, not necessarily self-adjoint, with both natural and Dirichlet boundary conditions, and also give a new alternative numerical method for calculating both the principal eigenvalue and corresponding eigenvector in the case of natural boundary conditions. The principal eigenvalue, if appropriate sign changes are made, determines the stability of equilibrium solutions to certain second order nonlinear partial differential equations. The corresponding eigenvector enables one to determine the first approximation of the solution of the nonlinear equation to variations of the initial condition from the equilibrium solution. These nonlinear equations are important in the applications. For these reasons it is important to have these characterizations of the principal eigenvalue and eigenvector.

Our method converts the determination of the eigenvalue and eigenvector to determining the solution of a stationary stochastic control

problem. This latter problem is solved and from it a numerical scheme arises naturally. This method appears to have applications in solving other problems.

Secondly, progress has been made on determining the asymptotic behavior of the principal eigenvalue for some singularly perturbed eigenvalue problems as a small nuisance parameter tends to zero. The principal eigenvalue is the optimal value for a singularly perturbed stationary stochastic control problem. We are thus able to determine the asymptotic behavior of the optimal value of certain stationary stochastic control problems.

Long-Term Effects of Noise on Dynamical Systems

Stochastically perturbed limit cycles, J. Applied Probability 15, 311-320 (1978).

An important question in the stability and control of stochastic systems is the determination of the limiting long-time behavior of the system using a fixed control. This work answered that question in he case of a stochastic system perturbed by a small additive noise term where the control is such that the corresponding deterministic system possesses a stable limit cycle.

It is shown that in he limit of large time the stochastic system is near the limit cycle. This is a stability result. Moreover, one can compute approximately at which portions of the limit cycle one is most likely to be found. Further various stationary average can be computed.

These results will be of use in designing approximate controls for stationary stochastic control systems. For a detailed discussion of these results, see the completed above paper.

Asymptotic Properties of Nonlinear - Diffusion Equations.

A nonlinear diffusion problem arising in plasma physics, (with J. Berryman), Phys. Rev. Letters, 40 (1978), 1720-1722.

A nonlinear generalization of the heat equation arising in plasma physics, (with J. Berryman), in Applied Nonlinear Analysis, 61-66, V. Lakshmikantham (Editor), Academic Press (1979).

Evolution of a stable profile for a class of nonlinear diffusion equations with fixed boundaries, (with J. Berryman), J. Math. Phys. 19 (1978), 2476-2480.

Stability of the separable solution for fast diffusion, (with J. Berryman), Arch. Rat. Mech. Anal., 74(1980), 379-388.

A technique is developed for studying the asymptotic behavior of certain classes of systems arising in the applications which are governed by nonlinear parabolic partial differential equations. The techniques involved the construction of an appropriate Liapunov type function in the spirit of the Liapunov approach for ordinary differential equations. It is shown that under certain conditions the solution of the nonlinear parabolic equation evolves toward an appropriate separable solution of the parabolic equation. The initial condition only determines the asymptotic amplitude of the separable solution.

The first paper above is a summary of the application of this general method to a problem in plasma physics which arises in studying the diffusion of ionized gas.

A potential weakness of the approach utilized above has been the inability to obtain decay rates of the solution to the equilibrium position and the inability to verify the higher order approximations which we have been able to construct formally. We have worked, without success, in this area.

Additionally, we have studied without success the uniqueness question of the limiting equilibrium position. The limiting position is described by the equation $\Delta u + u^{\alpha} = 0$, $u = 0$ on $\partial\Omega$, $\alpha > 1$.

PUBLICATIONS WITH SUPPORT OF THE AFOSR GRANT 77-3286

Stochastically perturbed limit cycles, J. Appl. Prob. 15(1978), 311-320.

A minimum principle for the smallest eigenvalue for second order linear elliptic equations with natural boundary conditions, Comm. Pure Appl. Math. 31(1978), 509-519.

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Evolution of a stable profile for a class of nonlinear diffusion equations with fixed boundaries(with J. Berryman), J. Math. Phys. 19(1978), 2476-2480.

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Stability of the separable solution for fast diffusion(with J. Berryman), Arch. Rat. Mech. Anal., 74(1980) 379-388.

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A nonlinear generalization of the heat equation arising in plasma physics (with J. Berryman), in Applied Nonlinear Analysis, 61-66, V. Lakshmikantham, Editor, Academic Press (1979).

INVITED LECTURES

CHARLES J. HOLLAND

Brown University, Providence,R.I., November 16,1977, "A Minimum Principle for Linear Second Order Elliptic Equations"

International Conference on Stochastic Analysis, Northwestern University, Evanston, Illinois, April 10-14,1978, "The Principal Eigenvalue for Linear Second Order Elliptic Equations With Natural Boundary Conditions"

International Conference on Nonlinear Analysis, University of Texas at Arlington, Arlington, Texas, April 20-22, 1978, "A Nonlinear Generalization of the Heat Equation Arising in Plasma Physics"

Brown University, Providence, R. I., May 9,1978, "Stability of the Separable Solution for Fast Diffusion"

Regional Conference on Stochastic Dynamical Systems-Theory and Applications, Clemson University, Clemson, South Carolina, July 17-21,1979, "Probabilistic Methods in Partial Differential Equations"

University of Georgia, Athens, GA, February 27, 1981, "Nonlinear Diffusion"

San Diego State University, San Diego, CA, April 2, 1981, "Nonlinear Diffusion"

Biography and Bibliography of Charles J. Holland

Education

B.S.	Georgia Institute of Technology	1968
M.S.	Georgia Institute of Technology	1969
Ph.D.	Brown University	1972

Professional Experience

Teaching Assistant	Georgia Institute of Technology	1967-69
Teaching Assistant	Brown University	1970-71
Research Assistant	Brown University	1971-72
Assistant Professor	Purdue University	1972-77
Associate Professor	Purdue University	1977-
Visiting Member	Courant Institute, New York Univ.	1976-78

Publications

1. A numerical technique for small noise stochastic control problems, J. Optimization Theory Appl., 13 (1974), 74-93.

2. Small noise open loop control, SIAM J. Control, 12 (1974), 380-388.

3. Ergodic expansions in small noise problems, J. Differential Equations, 16 (1974), 81-88.

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19. A new energy characterization of the smallest eigenvalue of the Schrödinger equation, Comm. Pure Appl. Math., 30 (1977), 755-765.

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24. Evolution of a stable profile for a class of nonlinear diffusion equations with fixed boundaries, (with J. Berryman), J. Math. Phys. 19 (1978), 2476-2480.

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27. An approximation technique for small noise open loop control problems, Optimal Control Appl. and Methods 2(1981), 89-94.

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The research was concentrated in the following areas: (1) approximation procedures for open loop stochastic control problems; (2) stationary stochastic control and eigenvalue problems; (3) long-term effects of noise on dynamical systems; and (4) asymptotic properties of nonlinear-diffusion equations. The first three areas are relevant to the control of stochastic systems. The fourth area deals with stability properties and the long term behavior of (CONTINUED ON REVERSE)

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ITEM #20, CONTINUED:

systems described by nonlinear partial differential equations. No progress was made in the area of nonlinear filtering, although it should be remarked that a transformation in area (2) above was developed and utilized independently by W. Fleming, also under AFOSR support, to study the unnormalized Zakai equation arising in nonlinear filtering.

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